

Applied mathe - I

D: PH (April Exam) 181

Con. 6865-13.

(REVISED COURSE)

GS-5103

(3 Hours)

[Total Marks : 80

- N.B.** (1) Question No. 1 is compulsory.
 (2) Attempt any **three** questions from Question Nos. 2 to Questions No. 6
 (3) **Figures** to the **right** indicate **full** marks.

1. (a) If $\cos hx = \sec \theta$ prove that $x = \log (\sec \theta + \tan \theta)$. 3
 (b) If $u = \log (x^2 + y^2)$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ 3
 (c) If $x = r \cos \theta$, $y = r \sin \theta$. Find $\frac{\partial(x, y)}{\partial(r, \theta)}$. 3
 (d) Expand $\log (1 + x + x^2 + x^3)$ in powers of x upto x^6 . 3
 (e) Show that every square matrix can be uniquely expressed as sum of a symmetric and a Skew-symmetric matrix. 4
 (f) Find n^{th} order derivative of $y = \cos x \cdot \cos 2x \cdot \cos 3x$. 4
2. (a) Solve the equation $x^6 - i = 0$. 6
 (b) Reduce matrix A to normal form and find its rank where :- 6

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

- (c) State and prove Euler's theorem for a homogeneous function in two variables and 8

hence find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ where $u = \frac{\sqrt{x} + \sqrt{y}}{x + y}$

3. (a) Determine the values of λ so that the equations $x + y + z = 1$; $x + 2y + 4z = \lambda$; $x + 4y + 10z = \lambda^2$ have a solution and solve them completely in each case. 6
 (b) Find the stationary values of $x^3 + y^3 - 3axy$, $a > 0$. 6
 (c) Separate into real and imaginary parts $\tan^{-1}(e^{i\theta})$. 8

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4. (a) If $x = u \cos v$, $y = u \sin v$ 6

Prove that $\frac{\partial(x, y)}{\partial(u, v)} \cdot \frac{\partial(u, v)}{\partial(x, y)} = 1$.

- (b) If $\tan [\log (x + iy)] = a + ib$, prove that $\tan [\log (x^2 + y^2)] = \frac{2a}{(1 - a^2 - b^2)}$ where 6

$a^2 + b^2 \neq 1$.

- (c) Using Gauss-Siedel iteration method, solve 8

$$10x_1 + x_2 + x_3 = 12$$

$$2x_1 + 10x_2 + x_3 = 13$$

$$2x_1 + 2x_2 + 10x_3 = 14$$

upto three iterations.

5. (a) Expand $\sin^7 \theta$ in a series of sines of multiples of θ . 6

- (b) Evaluate $\lim_{x \rightarrow 0} \frac{(x^x - x)}{(x - 1 - \log x)}$ 6

- (c) If $y^{1/m} + y^{-1/m} = 2x$, prove that 8

$$(x^2 - 1) y_{n+2} + (2n + 1) x y_{n+1} + (n^2 - m^2) y_n = 0.$$

6. (a) Examine the following vectors for linear dependence/Independence. 6

$$X_1 = (a, b, c), X_2 = (b, c, a), X_3 = (c, a, b) \text{ where } a + b + c \neq 0.$$

- (b) If $z = f(x, y)$, $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that 6

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$$

- (c) Fit a straight line to the following data and estimate the production in the year 1957. 8

Year :	1951	1961	1971	1981	1991
Production in the Thousand tons :	10	12	08	10	13