FE-(SEM I) (REV) (CBSGS) May 2014
applied mathematics I
13/05/14

(REVISED COURSE) Q P Code : **NP-17690**

(3 Hours) [Total Marks : 80

N.B. : (1) Questions **No. 1** is **compulsory.**
(2) **Attempt** any **three** from the remaining questions.
(3) Assume suitable data if necessary.

1. (a) Prove that $\mathrm{Sech}^{-1}(\sin\theta) = \log\left(\cot\frac{\theta}{2}\right)$ **3**

(b) If $x = \cos\theta - r\sin\theta$, $y = \sin\theta + r\cos\theta$ **3**

prove that $\frac{dr}{dx} = \frac{x}{r}$

(c) If $x = e^{v}\sec u$, $y = e^{v}\tan u$ **3**

find $J\left(\frac{u, v}{x, y}\right)$

(d) If $y = \sin px + \cos px$ **3**

Prove that $y_n = p^n[1 + (-1)^n \sin)2px)]^{\frac{1}{2}}$

(e) Find the series expansion of $\log(1+x)$ in powers of x. Hence prove that **4**

$\log x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 \ldots\ldots$

(f) If 'A' is skew-symmetric matrix of odd order then prove that it is singular. **4**

2. (a) Show that the roots of the equation $(x+1)^6 + (x-1)^6 = 0$ are given by **6**

$-i\cot\left(\frac{2n+1}{12}\right)\pi, n = 0,1,2,3,4,5.$

(b) Find two non-singular matrices P & Q such that PAQ is in normal form where **6**

$$A = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 1 & 4 & -5 \\ -1 & -5 & -5 & 7 \end{bmatrix}$$

Also find rank of A.

(c) If $x + y = 2e^{\theta}\cos\phi$, $x - y = 2ie^{\theta}\sin\phi$ & u is a function of x & y the prove that **8**

$$\frac{\partial^2 u}{\partial\theta^2} + \frac{\partial^2 u}{\partial\varphi^2} = 4xy\frac{\partial^2 u}{\partial x\partial y}$$

3. (a) Find the value of λ for which the equations $x_1+2x_2+x_3=3$, $x_1+x_2+x_3=\lambda$, $3x_1+x_2+3x_3=\lambda^2$ has a solution & solve them completely for each value of λ. **6**

(b) Divide 24 into three parts such that the product of the first, square of the second & cube of the third is maximum. **6**

(c) (i) If $\operatorname{cosec}\left(\frac{\pi}{4}+ix\right)=u+iv$ prove that $(u^2+v^2)^2=2(u^2-v^2)$ **4**

(ii) Prove that $\tan\left(i\log\left(\frac{a-ib}{a+ib}\right)\right)=\frac{2ab}{a^2-b^2}$ **4**

4. (a) Show that $\frac{\partial(u,v)}{\partial(x,y)}=6r^3\sin 2\theta$ given that $u=x^2-y^2$, $v=2x^2-y^2$ & $x=r\cos\theta$, $y=r\sin\theta$. **6**

(b) If $\alpha=1+i$, $\beta=1-i$ & $\cot\theta=x+1$ prove that $(x+\alpha)^n+(x+\beta)^n=(\alpha+\beta)\cos n\theta\operatorname{cosec}^n\theta$. **6**

(c) Using Gauss-seidel method, solve the following system of equations upto 3rd iteration. **8**

$5x-y=9$

$-x+5y-z=4$

$-y+5z=-6$

5. (a) Using De-Moivr's theorem, prove that **6**

$$\frac{\sin 6\theta}{\sin\theta}=16\cos^4\theta-16\cos^2\theta+3$$

(b) Expland $\frac{x}{e^x-1}$ in powers of x. **6**

Hence prove that $\frac{x}{2}\left[\frac{e^x+1}{e^x-1}\right]=1+\frac{1}{12}x^2-\frac{1}{720}x^4+\ldots$

(c) If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ **8**

prove that $(1-x^2)y_{n+2}-(2n+3)xy_{n+1}-(n+1)^2y_n=0$. Hence find $y_n(0)$

6. (a) Examine the linear dependence or independence of vectors (1, 2, -1, 0), (1, 3, 1, 3), (4, 2, 1, -1) & (6, 1, 0, -5) **6**

(b) If $u = f\left(\frac{x-y}{xy}, \frac{z-x}{xz}\right)$ prove that **6**

$$x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$$

(c) (i) Fit a straight line to the following data with x - as independent variable. **4**

X	:	1965	1966	1967	1968	1969
Y	:	125	140	165	195	200

(ii) Evaluate $\lim_{x \to 0} (1 + \tan x)^{\cot x}$ **4**