

QP Code : 1002

(REVISED COURSE)

(3 Hours)

[Total Marks : 80

- N. B. : (1) Question No. 1 is compulsory.
 (2) Answer any three questions from remaining.
 (3) Assume suitable data if necessary.

1. (a) If $\tan \frac{x}{2} = \tanh \frac{u}{2}$ then S.T. 3

$$u = \log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$$

(b) If $u = x^y$ find $\frac{\partial^3 u}{\partial x \partial y \partial x}$ 3

(c) If $ux = yz, vy = zx, wz = xy$ 3

$$\text{find } J \left(\frac{u, v, w}{x, y, z} \right)$$

(d) If $y = (x-1)^n$ then P.T. $y + \frac{y_1}{1!} + \frac{y_2}{2!} + \frac{y_3}{3!} + \dots + \frac{y_n}{n!} = x^n$ 3

(e) P.T. $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$ 4

(f) Express the matrix A as sum of Hermitian and skew Hermitian matrix where 4

$$A = \begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$$

2. (a) Solve $x^7 + x^4 + i(x^3 + 1) = 0$ 6

(b) Reduce the matrix A to normal form and hence find its rank where 6

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 4 & 3 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

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- (c) State and prove Euler's theorem for three variables and hence find 8

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} \text{ where}$$

$$u = \frac{x^3 y^3 z^3}{x^3 + y^3 + z^3}$$

3. (a) Solve the following system of equations 6

$$2x - 2y - 5z = 0$$

$$4x - y + z = 0$$

$$3z - 2y + 3z = 0$$

$$x - 3y + 7z = 0$$

- (b) Find the maximum and minimum values 6

$$\text{of } x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$$

- (c) Separate into real and imaginary parts of $\tanh^{-1}(x + iy)$ 8

4. (a) If $u = 2xy$, $v = x^2 - y^2$ and 6

$$x = r \cos \theta, y = r \sin \theta \text{ then find}$$

$$\frac{\partial(u, v)}{\partial(r, \theta)}$$

- (b) If $i^{i^{i^{\dots \infty}}} = A + iB$, prove that 6

$$\left(\frac{\pi A}{2}\right) = \frac{B}{A} \text{ and } A^2 + B^2 = e^{-\pi B}$$

- (c) Solve by crouts method the system of equations 8

$$3x + 2y + 7z = 4$$

$$2x + 3y + z = 5$$

$$3x + 4y + z = 7$$

5. (a) By using De Moivre's theorem 6

$$\text{Express } \frac{\sin 7\theta}{\sin \theta} \text{ in powers of } \sin \theta \text{ only.}$$

- (b) By using Taylor's series expand $\tan^{-1} x$ in positive powers of $(x-1)$ upto first four non-zero terms. 6

- (c) If $y = \sin[\log(x^2 + 2x + 1)]$ prove that 8

$$(x+1)^2 y_{n+2} + (2n+1)(x+1) y_{n+1} + (n^2 + 4)y_n = 0$$

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6. (a) Determine linear dependence or independence of vectors 6

$$x_1 = [1, 3, 4, 2] \quad x_2 = [3, -5, 2, 6]$$

$x_3 = [2, -1, 3, 4]$ and if dependent find the relation between them.

- (b) If $u = x^2 - y^2$, $v = 2xy$ and $z = f(u, v)$ prove that 6

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 4\sqrt{u^2 + v^2} \left[\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2 \right]$$

- (c) (i) Evaluate $\lim_{x \rightarrow 0} \frac{\sin x \cdot \sin^{-1} x - x^2}{x^6}$ 4

(ii) Fit straight line to the following data 4

$(x, y) = (-1, -5), (1, 1), (2, 4), (3, 7), (4, 10)$

Estimate y when $x = 7$
