FE sem II All Branches

A. Maths 22/11/2012

V-L-II-Hf-Ex-12-B-40

Con. 8824-12.

KR-3531

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** from remaining **six** questions.

1. (a) Using Taylor's series find $y(0{\cdot}4)$ where $\frac{dy}{dx} = 1 + xy$ with $y(0) = 2$. 3

(b) Find the Complementary function of $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$. 3

(c) Evalaute $\int_{-1}^{1}\int_{0}^{1-x} x^{1/3} y \, dx \, dy$. 3

(d) Evaluate $\int_0^2\int_0^2\int_0^{yz} xyz \, dxdydz$. 3

(e) Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta = \pi$. 4

(f) Using Euler's method solve $\frac{dy}{dx} = x + y$, $y(0) = 1$ find the value of y at $x = 1$, taking $h = 0{\cdot}2$. 4

2. (a) Evalaute $\int_0^1 \sqrt{1-\sqrt{x}}\, dx \int_0^{\frac{1}{2}} \sqrt{2y - 4y^2}\, dy = \frac{\pi}{30}$. 6

(b) Use Runge-Kutta method of fourth order to solve :— 6

$\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$ find $y(0{\cdot}2)$ with $h = 0{\cdot}1$.

(c) Solve $\frac{dy}{dx} + x \sin 2y = x^3\cos^2 y$. 8

3. (a) Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. 6

(b) Solve $\left(D^2 - 1\right)y = \frac{2}{1+e^x}$ using variation of Parameter. 6

(c) Evaluate $\int_0^{\pi/2} \frac{dx}{1 + a\cos^2 x}$ and hence deduce that $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{(3 + \cos^2 x)^2}\, dx = \frac{\pi\sqrt{3}}{96}$. 8

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4. (a) Solve $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$. 6

(b) $(1+2x)^2\frac{d^2y}{dx^2}-2(1+2x)\frac{dy}{dx}-12y=6x.$ 6

(c) Solve $\frac{d^2y}{dx^2}+2y=x^2e^{3x}+e^x-\cos 2x.$ 8

5. (a) In a circuit containing inductance L, resistance R, and voltage E, the current i is given by $L\frac{di}{dt}+Ri=E$. find the current i at time t, at $t=0$, $i=0$, L, R, E are constants. 6

(b) Change the order of integration $\int_0^a \int_{\sqrt{a^2-y^2}}^{y+a} f(x,y)\,dx\,dy.$ 6

(c) Evaluate $\iiint \frac{dx\,dy\,dz}{\sqrt{a^2-x^2-y^2-z^2}}$ over the volume of Sphere $x^2+y^2+z^2=a^2$. 8

6. (a) Find the length of the parabola $x^2=4y$ which lies inside the circle $x^2+y^2=6y$. 6

(b) Change into polar and evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\,dx\,dy}{\sqrt{x^2+y^2}}$. 6

(c) Evaluate $\iint_R xy\sqrt{1-x-y}\,dx\,dy$ over the area of the triangle formed by— 8

$x=0$, $y=0$, $x+y=1$.

7. (a) Change the order of integration and evaluate :— 6

$$\int_0^a \int_0^x \frac{dxdy}{(y+a)\sqrt{(a-x)(x-y)}}$$

(b) Find the area outside the circle $r=a$ and inside the cardioide $r=a(1+\cos\theta)$. 6

(c) Find the volume common to the cylinders $x^2+y^2=a^2$, $x^2+z^2=a^2$. 8

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