(Revised course)

(3 Hours) [Total Marks : 80]

N.B. : (1) Question No. **1** is **compulsory**.

(2) Answer any **three** questions from question nos. 2 to 6.

(3) **Figures** to the **right** indicate **full** marks.

(4) Programming Calculators are **not allowed**.

1. (a) Evaluate $\int_0^\infty x^2 \, 7^{-4x^2} \, dx$ **3**

    (b) Solve $(D^4+4)y = 0$ **3**

    (c) Prove that $E\nabla = \Delta = \nabla E$ **3**

    (d) Solve $(x + 2y^3)\dfrac{dy}{dx} = y$. **3**

    (e) Evaluate $\iint_R r^3 \, dr \, d\theta$ over the region between the circles $r = 2\sin\theta$, $r = 4\sin\theta$. **4**

    (f) Evaluate $\int_0^1 \int_y^{\sqrt{y}} \dfrac{x}{(1-y)\sqrt{y-x^2}} \, dy \, dx$ **4**

2. (a) Solve :- $(x^3y^4 + x^2y^3 + xy^2 + y) \, dx + (x^4y^3 - x^3y^2 - x^2y + x) \, dy = 0$ **6**

    (b) Change the order of integral and hence evaluate $\int_0^5 \int_{2-x}^{2+x} dx \, dy$ **6**

    (c) Prove that $\int_0^1 \dfrac{x^2}{\sqrt{1-x^4}} \, dx \int_0^1 \dfrac{dx}{\sqrt{1+x^4}} = \dfrac{\pi}{4\sqrt{2}}$ **8**

3. (a) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dx \, dy \, dz$. **6**

    (b) Find the area of one loop of the lemniscate $r^2 = a^2 \cdot \cos 2\theta$ **6**

    (c) Solve $(D^3 + 2D^2 + D)y = x^2 e^{3x} + \sin^2 x + 2^x$. **8**

4. (a) Show that the length of arc of the parabola $y^2 = 4ax$ cut off by the line $3y = 8x$ is **6**

    $a\left(\log 2 + \dfrac{15}{16}\right)$

    (b) Using the method of variation of parameters solve $\dfrac{d^2y}{dx^2} + 4y = \tan 2x$. **6**

    (c) Compute $y(0.2)$ given $\dfrac{dy}{dx} + y + xy^2 = 0$, $y(0) = 1$ by taking $h = 0.1$ using Runge-Kutta **8**
    method of fourth order correct to 4 decimals.

[**TURN OVER**

5. (a) Solve $\dfrac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1$.   6

    (b) Solve $\dfrac{dy}{dx} - 2y = 3e^x$, $y(0) = 0$ using Taylor series method. Find approximate value   6
         of y for x = 1 and 1.1.

    (c) Evaluate $\displaystyle\int_0^6 \dfrac{dx}{1+x}$ using   8

         (i)   Trapezoidal rule

         (ii) Simpson's $\left(\dfrac{1}{3}\right)^{rd}$ rule and

         (iii) Simpson's $\left(\dfrac{3}{8}\right)^{th}$ rule.

         Compare result with exact values.

6. (a) The current in a circuit containing an inductance L, registance R and voltage   6
         E sin wt is given by

         $L\dfrac{di}{dt} + Ri = E \sin wt$

         If i = 0 at t = 0, find i.

    (b) Evaluate $\displaystyle\iint_R e^{2x-3y} \, dxdy$ over the triangle bounded by x + y = 1, x = 1, y = 1.   6

    (c) (i)   Find the volume of solid bounded by the surfaces $y^2 = 4ax$, $x^2 = 4ay$ and the   4
               planes Z = 0, Z = 3.

         (ii) Change to polar co-ordinates and evaluate   4

               $\displaystyle\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \dfrac{dxdy}{\sqrt{a^2 - x^2 - y^2}}$