

(REVISED COURSE)

QP Code : 11859

(3 Hours)

Total Marks : 80

- N. B. : (1) Question No.1 is compulsory.
 (2) Attempt **any three** questions of the remaining five.
 (3) **figures** to the right indicate **full marks**.

1. (a) Evaluate $\int_0^2 x^4 (8-x^3)^{-1/3} dx$ 3
- (b) Solve $\frac{d^4 y}{dx^4} + 2\frac{d^2 y}{dx^2} + y = 0$ 3
- (c) Prove that $E = 1 + \Delta = e^{4D}$ 3
- (d) Solve $[x\sqrt{x^2 + y^2} - y]dx + [y\sqrt{x^2 + y^2} - x]dy = 0$ 3
- (e) Change to polar coordinates and evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} \frac{x}{\sqrt{x^2 + y^2}} dy dx$ 4
- (f) Evaluate $\int_0^1 \int_0^x e^{x+y} dy dx$ 4
2. (a) Solve $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ 6
- (b) Change the order of integration and evaluate $\int_0^a \int_{y^2/a}^y \frac{y}{(a-x)\sqrt{ax-y^2}} dx dy$ 6
- (c) Prove that $\int_0^\infty \cos \lambda x (e^{-ax} - e^{-bx}) dx = \frac{1}{2} \log \left(\frac{b^2 + \lambda^2}{a^2 + \lambda^2} \right)$, $a > 0, b > 0$ using DUIS 8
 rule
3. (a) Evaluate $\iiint \frac{dx dy dz}{x^2 + y^2 + z^2}$ throughout the volume of the sphere 6
 $x^2 + y^2 + z^2 = a^2$
- (b) Find the area common to the cardioids $r = a(1 + \cos \theta)$ and $r = a(1 - \cos \theta)$. 6
- (c) Apply the method of variation of parameters to solve 8
 $\frac{d^2 y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} \sec^2 x$
4. (a) Find the length of one arc of the cycloid $x = a(\theta - \sin \theta)$ and $y = a(1 + \cos \theta)$ 6
- (b) Solve $\frac{d^2 y}{dx^2} + 2y = x^2 e^{3x} + e^x \cos x$ 6

- (c) Apply Runge- kutla method of fourth order to find an approximate value of y at $x = 1.2$ if $\frac{dy}{dx} = x^2 + y^2$, given that $y = 1.5$ when $x = 1$ choosing $h = 0.1$ 8
5. (a) Solve $[xy^2 - e^{1/x^3}] dx - yx^2 dy = 0$ 6
- (b) If y satisfies the equation $\frac{dy}{dx} = x^2 y - 1$ and with $y = 1$ when $x = 0$, using Taylor's series method for y about $x = 0$, find y when $x = 0.1$ and $x = 0.2$ 6
- (c) Compute the value of the definite integral $\int_{-1}^1 \frac{dx}{1+x^2}$ by using 8
- (i) Trapezoidal rule
- (ii) Simpson's $\left(\frac{1}{3}\right)^{\text{rd}}$ rule
- (iii) Simpson's $\left(\frac{3}{8}\right)^{\text{th}}$ rule. Compare result with exact values.
6. (a) A radial displacement 'u' in rotating a disc at a distance 'r' from the axis in given by $\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} + kr = 0$. Find the displacement given $u = 0$ when $r = 0$ and $r = a$ 6
- (b) Evaluate $\iint x^2 dx dy$ over the region bounded by $xy = a^2$, $x = 2a$, $y = 0$ and $y = x$ in the first quadrant. 6
- (c) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ 8
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