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DEGREE & DIPLOMA ENGINEERING CLASSES

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MODEL QUESTION PAPER – I (May 2016)

F. E. SEMESTER – II

APPLIED MATHEMATICS – II (REVISED)

[Time: 3 hours] **[Marks: 80]**

N. B.: 1. Question No. 1 is compulsory.
2. Attempt any three from remaining five questions.
3. Figure to the right indicates full marks.

1.

a. Show that $\int_0^\infty \sqrt{y}.e^{-y^2}dy.\int_0^\infty \frac{e^{-y^2}}{\sqrt{y}}dy = \frac{\pi}{2\sqrt{2}}$ **(3)**

b. Solve: $\left(D^2+6D+9\right)y = \sinh 3x$ **(3)**

c. Prove that: $\Delta \log f(x) = \log\left\{1+\frac{\Delta f(x)}{f(x)}\right\}$ **(3)**

d. Express the following integrals in polar co-ordinates and hence evaluate: $\int_0^1\int_0^x (x+y)\,dy\,dx$ **(4)**

e. Solve the given differential equation: $\frac{dy}{dx} = \frac{\tan y - 2xy - y}{x^2 - x\tan^2 y + \sec^2 y}$ **(4)**

f. Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\,dx\,dy$ **(3)**

2.

a. Solve the given differential equation: $\frac{dr}{d\theta} = \frac{r\sin\theta - r^2}{\cos\theta}$ **(6)**

b. Change the order of Integration and evaluate: $\int_0^a\int_0^x \frac{\sin y}{\sqrt{(a-x)(x-y)(4-5\cos y)^2}}\,dy\,dx$ **(6)**

c. Assuming the validity of differentiation under integral sign, prove that : **(8)**

$$\int_0^\infty \frac{e^{-x}-e^{-ax}}{x\cos x}dx = \frac{1}{2}\log\left(\frac{a^2+1}{2}\right), (a>0)$$

3.

a. Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dx\, dy\, dz$ **(6)**

b. Show that the area between the curves $y = ax^2$ and $y = 1 - \frac{x^2}{a}, a > 0$ is $\frac{4}{3}\sqrt{\frac{a}{a^2+1}}$ **(6)**

c. Solve the following using the variation of parameters: $\left(D^2 + 6D + 9\right) y = e^{-3x} \frac{1}{x^2}$ **(8)**

4.

a. Find the length of the cardioide $r = a\left(1 - \cos\theta\right)$ lying **(6)**

i) outside the circle $r = a\cos\theta$

ii) inside the circle $r = a\cos\theta$

b. Solve the given differential equation: $\left(D^2 - 4\right) y = x^2 e^{3x}$ **(6)**

c. Using Runge-Kutta fourth order method, find the numerical solution at x = 0.6 for **(8)**
$\frac{dy}{dx} = \sqrt{x+y}, y\left(0.4\right) = 0.41$ assume step length h = 0.2.

5.

a. Solve the given differential equation: $\left(x\sec^2 t - x^2 \cos y\right) dy = \left(\tan y - 3x^4\right) dx$ **(6)**

b. Use Euler's modified method to find the values of y satisfying the equation $\frac{dy}{dx} = \log\left(x + y\right)$, for x = 1.2 and x = 1.4 correct to three decimals by taking h = 0.2 and y(1) = 2. **(6)**

c. Solve $\int_1^4 \left(e^x + x^3 - 2x + 1\right) dx$ by using: **(8)**

i) Trapezoidal rule

ii) Simpson's $\frac{1}{3}$rd rule.

iii) Simpson's $\frac{3}{8}$th rule, Assume 12 divisions.

6.

a. The radial displacement u in a rotating dice at a distance r from the axis is given by **(6)**
$r^2 \frac{d^2u}{dr^2} + r\frac{du}{dr} - u + kr^3 = 0$ where k is a constant. Find the displacement if u = 0 when r = a and r = 0.

b. Evaluate $\iint_R \frac{\tan^{-1} y}{\left(1 + y^2\right)\sqrt{\left(1-x\right)\left(x-y\right)}} dx\, dy$ where R is the region bounded by straight line **(6)**
$x = 1, x = y, y = 0$

c. Find the volume cut off from the sphere $x^2 + y^2 + z^2 = a^2$ by the cone $x^2 + y^2 = z^2$ **(8)**

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...by Dr. A. K. Pathak