MODEL QUESTION PAPER (May 2016)

S. E. SEMESTER – IV (REVISED)

APPLIED MATHEMATICS – IV (EXTC/ETRX/BIOM/INST/ELEC)

[Time: 3 hours] **[Marks: 80]**

N. B.: 1. Question no. 1 is compulsory.

2. Attempt any three question out of remaining five question .

3. Figure to right indicate full marks.

1.

a) Evaluate: $\oint_C \frac{\sin^6 z}{\left(z-\left(\frac{\pi}{2}\right)\right)^3} dz$ around the circle $|z| = 2$. **(5)**

b) For a matrix, $A=\begin{bmatrix}1 & -2 & 0\\ 1 & 2 & 2\\ 1 & 2 & 3\end{bmatrix}$, Find characteristic equation and Eigen values of A^2. **(5)**

c) Find approximate solution of the problem for the extremum of the functional. $I=\int_0^1\left(xy+\frac{1}{2}y'^2\right)dx$ subject to the condition $y(0) = y(1) = 0$. **(5)**

d) Check whether the following are subspaces of R^3. $W=\{(x, y, z) \mid x^2 + y^2 + z^2 \le 1\}$ **(5)**

2.

a) Find the extremal of $\int_{x0}^{x1}(16y^2 - y''^2 + x^2)dx$ **(6)**

b) If $A=\begin{bmatrix}2 & 3\\ -3 & -4\end{bmatrix}$ prove that $A^{100}=\begin{pmatrix}-299 & -300\\ 300 & 301\end{pmatrix}$. **(6)**

c) Find all possible Laurent's expansions of the function $f(z) = \frac{7z-2}{z(z-2)(z+1)}$ about $z = -1$. **(8)**

3.

a) Using Cauchy's integral formula evaluate $\oint_C \frac{e^{5z}}{(z+i)^4} dz$ where C is $|z| = 3$. **(6)**

b) Find the characteristic equation of the matrix : $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$ and hence find:

(6)

i) The matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$.

ii) A^{-1}.

c) Find the singular value decomposition of the following matrix. **(8)**

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$$

4.

a) Let R^3 have the Euclidean inner product. Use gram-schmidt process to transform the basis (u_1, u_2, u_3) into an Orthonormal basis where u_1=(1, 1, 1), u_2=(-1, 1, 0), u_3=(1, 2, 1). **(6)**

b) Show that the matrix, $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$ is derogatory and find its minimum polynomial. **(6)**

c) i) Prove: $\int_0^{\infty} \frac{dx}{x^4+1} = \frac{\pi\sqrt{2}}{4}$ **(4)**

ii) Show that $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta = \frac{\pi}{6}$. Using Cauchy's Residue Theorem. **(4)**

5.

a) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show that for every integer $n \geq 3, A^n = A^{n-2} + A^2 - I$. Hence determine A^{50}. **(6)**

b) Prove that V which is a set of all the points lying on the line passing through the origin in R^2 is vector space under standard addition and scalar multiplication. OR

$V = \{(x, y) \mid ax + by = 0 \quad \text{in } R^2\}$.

Prove That V is a vector space under standard addition and scalar multiplication. **(6)**

c) Show that the matrix, $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable. Also find the diagonal form and diagonalizing matrix P. **(8)**

6.

a) Are the vector's $X_1 = [1,3,4,2]; X_2 = [3,-5,2,6]; X_3 = [2,-1,3,4]$ linearly dependent? If so express X_1 as a linear combination of the other. **(6)**

b) Evaluate $\int_0^{1+i} (x^2 + iy)\, dz$ along the path i) y = x ii) $y = x^2$. Is the line integral independent of path? Explain. **(6)**

c) Reduce the quadratic form, by congruent reduction, $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$. to the sum of squares find the corresponding linear transformation find rank, index and signature. **(8)**

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...by Dr. A. K. Pathak