MODEL QUESTION PAPER (May 2016)
S. E. SEMESTER – IV (REVISED)
APPLIED MATHEMATICS – IV (MECH/CIVIL/MECHOTRONICS/AUTO/PROD)

[Time: 3 hours] **[Marks: 80]**

N. B.: 1. Question no. 1 is compulsory.
2. Attempt any three question out of remaining five question.
3. Figure to right indicate full marks.

1.

a) Ten individuals are chosen at random from a population and their heights are found to be in inches 63, 63, 64, 65, 66, 69, 69, 70, 70, 71. Discuss the suggestion that the mean height of the universe is 65. **(5)**

b) Evaluate $\oint_C (y-\sin x)dx+(\cos x)dy$ where C is a triangle formed by the lines $y=0, 2x=\pi \text{ and } 2x=\pi y$. **(5)**

c) If $A=\begin{bmatrix} \pi & \frac{\pi}{4} \\ 0 & \frac{\pi}{2} \end{bmatrix}$, find cos A. **(5)**

d) For a Binomial variate if mean = 3 and $15P(X=0)=2P(X=1)$, find $P(X=5)$. **(5)**

2.

a) Investigate the association between the darkness of eye colour in father and son from the following data. **(6)**

	Dark	Not Dark	Total
Dark	48	90	138
Not Dark	80	782	862
Total	128	872	1000

b) In partially destroyed laboratory record of an analysis of correlation data, the following results only are legible:variance of X = 9 regression equations : $8X-10Y+66=0, 40X-18Y=214$ **(6)**

What were (i) the mean values of X and Y
(ii) the correlation coefficients between X and Y and
(iii) the S.D. of Y?

c) Using the Lagrangian Multipliers solve the following N.L.P.P. **(8)**

Optimize $Z=2x_1^2+2x_2^2+2x_3^2-24x_1-8x_2-12x_3+196$

Subject to $x_1+x_2+x_3=11$

$x_1, x_2, x_3 \ge 0$

3.

a) Find the directional derivative of $\phi = e^{2x}\cos yz$ at the origin in the direction of tangent to the curve x = a sin t, y = a cos t, z = a t at = $\frac{\pi}{4}$. **(6)**

b) Find the characteristic equation of the matrix : $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find: **(6)**

i) The matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$.

ii) A^{-1}.

c) Using appropriate theorems evaluate the following integrals $\iint_S \left(\nabla \times \overline{F}\right).d\overline{s}$ over the hemisphere $x^2+y^2+z^2=16$ above xy plane where $\overline{F}=\left(x^2+y-4\right)\hat{i}+3xy\,\hat{j}+\left(2xz+z^2\right)\hat{k}.$ **(8)**

4.

a) Seven dice are thrown 729 times. How many times do you expect at least four dice to show three or five? **(6)**

b) Show that the matrix, $A=\begin{bmatrix}1 & 0 & 0\\ 1 & -1 & 0\\ 1 & 0 & -1\end{bmatrix}$ is derogatory and find its minimum polynomial. **(6)**

c) Show that the matrix, $A=\begin{bmatrix}-9 & 4 & 4\\ -8 & 3 & 4\\ -16 & 8 & 7\end{bmatrix}$ is diagonalizable. Also find the diagonal form and diagonalizing matrix P. **(8)**

5.

a) Prove that $\overline{F}=(2xy+z)i+\left(x^2+2yz^3\right)j+\left(3y^2z^2+x\right)k$ is irrotational. Find the scalar potential ϕ show that $\overline{F}=$ grad ϕ and evaluate $\int_A^B \overline{F}.\overline{dr}$ along the straight line joining A(1, 2, 0) to B(2, 2, 1). **(6)**

b) A continuous random variable 'x' has probability density function $f(x)$ given by

$$f(x) = \begin{cases} kx(1-x) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find k, mean, variance, S.D. and $P\left(|x-m|<\sigma\right)$, where m is mean and $\sigma$is the standard deviation of x. **(6)**

c) The marks obtained by a number of students in a certain subject are approximately normal with mean 65 and variance 25. If 3students are selected at random from this group, what is the probability that at least one of them would have scored above 75? **(8)**

6.

a) Evaluate $\iint_S \overline{F}.\hat{n}\,ds$ where $\overline{F}=2xy\,\hat{i}+yz^3\,\hat{j}+xz\,\hat{k}$ and S is the surface of the parallelepiped bounded by $x=0, y=0, x=0, z=0, x=2, y=1, z=3.$ **(6)**

b) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than the girls. **(6)**

c) Derive the optimal solution from the Kuhn–Tucker conditions, for the problem – **(8)**

Minimize $z=2x_1+3x_2-x_1^2-2x_2^2$.

Subject to the constraints: $x_1+3x_2 \le 6$

$5x_1+2x_2 \le 10$.

$x_1, x_2 \ge 0$.

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...by Dr. A. K. Pathak