MULUND	314, 3rd Floor, Shree Samarth Plaza, R. R. T. Road, Opp. Rly station, Mulund(W). Phone: **022 25621515, 9920030136.**
DADAR	C/o Kalpana classes, 205, Second floor, Pearl centre, Senapati Bapat Marg, Dadar (W). Phone: **9819830985.**
KALYAN	209, Subhalaxmi Shopping Centre, Shivaji Chowk, Agra Road, Kalyan(W). Ph No.:**0251 2202887, +91 99876 06730.**
PANVEL	Plot No. 13, Sector – 1, Road No. 14, Near Shanti Niketan Vidyalay New Panvel, Navi Mumbai, Mob No.:**+91 9004679516**.

MODEL QUESTION PAPER – II (Dec- 2015)

F. E. SEMESTER – I

APPLIED MATHEMATICS – I (REVISED)

[Time: 3 hours] **[Marks: 80]**

1.

a. If $y = \log(\tan x)$, prove that $\sinh ny = \frac{1}{2}\left(\tan^n x - \cot^n x\right)$. **(3)**

b. If $u = \log(\tan x + \tan y + \tan z)$, prove that $\frac{\partial u}{\partial x}\sin 2x + \frac{\partial u}{\partial y}\sin 2y + \frac{\partial u}{\partial z}\sin 2z = 2$. **(3)**

c. If $x = u^2 - v^2,\ y = 2uv$, find $\frac{\partial u}{\partial x},\ \frac{\partial v}{\partial x}$. **(3)**

d. Show that $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots$ **(4)**

e. Prove that every Hermitian matrix A can be expressed as B + iC where B is real symmetric and C is real skew-symmetric. **(4)**

f. Find the n^{th} derivative w.r.t. x of: $\frac{8x}{x^3 - 2x^2 - 4x + 8}$ **(3)**

2.

a. Show that the roots of $z^5 = 1$ can be written as $1, \alpha, \alpha^2, \alpha^3, \alpha^4$. Hence prove that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4) = 5$. **(6)**

b. Find non singular matrices P and Q such that PAQ is in normal form; hence find rank of A.

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 2 \\ 7 & 4 & 10 \\ 1 & 0 & 6 \end{bmatrix}$$ **(6)**

c. If $u = \sin^{-1}\sqrt{\frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}}$, show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}(13 + \tan^2 u)$. **(8)**

3.

a. If the following system has nontrivial solution, prove that $a+b+c=0$ *or* $a=b=c$.
$ax+by+cz=0, bx+cy+az=0, cx+ay+bz=0$. **(6)**

b. Discuss the stationary values of $u=x^2y-3x^2-2y^2-4y+3$. **(6)**

c. If $\log\cos(x-iy)=\alpha+i\beta$. Prove that $\alpha=\frac{1}{2}\log\left[\frac{\cosh 2y+\cos 2x}{2}\right]$ and find β. **(8)**

4.

a. For the transformation $x=e^u\cos v, y=e^u\sin v$, prove that $\frac{\partial(x,y)}{\partial(u,v)}\times\frac{\partial(u,v)}{\partial(x,y)}=1$ **(6)**

b. If $(a+ib)^p=m^{x+iy}$, prove that $\frac{y}{x}=\frac{2\tan^{-1}\left(\frac{b}{a}\right)}{\log(a^2+b^2)}$. **(6)**

c. Solve the following equations by Crout's method:

$4x+y-z=13,$
$3x+5y+2z=21,$
$2x+y+6z=14$ **(8)**

5.

a. Using De Moivre's theorems show that
$2(1+\cos 8\theta)=(x^4-4x^2+2)^2$ where $x=2\cos\theta$. **(6)**

b. Find the value of a and b such that $\lim_{x\to 0}\frac{a\sin^2 x+b\log\cos x}{x^4}=\frac{1}{2}$ **(6)**

c. If $y=e^{a\cos^{-1}x}$, show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$. Hence, find $y_n(0)$. **(8)**

6.

a. Examine whether the vectors $X_1=[3,1,1], X_2=[2,0,-1], X_3=[4,2,1]$ are linearly independent. **(6)**

b. If $\frac{x^2}{a^2+u}+\frac{y^2}{b^2+u}+\frac{z^2}{c^2+u}=1$. prove that $(u_x)^2+(u_y)^2+(u_z)^2=2(xu_x+yu_y+zu_z)$. **(6)**

c. Fit a straight line to the following data : **(8)**

Year (X)	1951	1961	1971	1981	1991
Production (Y) (1000 tons)	10	12	8	10	13

Also estimate the production in1957 & 1987.

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...by Dr. A. K. Pathak