GLORIOUS ACADEMY

Success is a process not an event, so come be part of process

DEGREE & DIPLOMA ENGINEERING CLASSES

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MODEL QUESTION PAPER – I (Dec- 2015)

F. E. SEMESTER – I

APPLIED MATHEMATICS – I

[Time: 3 hours] **[Marks: 80]**

1.

a. If tanh x = $\frac{1}{2}$, prove that sinh 2x = $\frac{4}{3}$ **(3)**

b. If u = x^y, show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$ **(3)**

c. If $u = \frac{y-x}{1+xy}$ and $v = \tan^{-1} y - \tan^{-1} x$, find $\frac{\partial(u,v)}{\partial(x,y)}$. **(3)**

d. Obtain the series of log (1 + x) and hence find the series of $\log_e \left(\frac{1+x}{1-x}\right)$. **(4)**

e. Prove that Diagonal elements of skew-Hermitian matrix are purely imaginary or zero. **(4)**

f. Find the n^{th} derivative w.r.t. x of: $2^x \sin^2 x \cos^3 x$. **(3)**

2.

a. Show that the roots of $(x + 1)^7 = (x - 1)^7$ are given by $-i \cot\frac{r\pi}{7}$, r = 1, 2, 3…6. Why is $r \neq 0$? **(6)**

b. Find rank of A by reducing it into the normal form, where $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$ (6)

c. State & prove **Euler's Theorem** for a function of two variables. Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ for $u = (8x^2 + y^2)(\log x - \log y)$. **(8)**

3.

a. Find for what values of λ and μ the simultaneous equations:

$x + y + z = 6$

$x + 2y + 3z = 10$

$x+2y+\lambda z=\mu$

have i) no. solution ii) a unique solution iii) infinite no. of solutions. **(6)**

b. Find all the stationary points of the function $x^3+3xy^2-15x^2-15y^2+72x$ examine whether the function is maximum or minimum at those points. **(6)**

c. Show that $\sin^{-1}(e^{i\theta}) = \cos^{-1}\sqrt{\sin\theta} + i\log\left[\sqrt{\sin\theta}+\sqrt{1+\sin\theta}\right]$. **(8)**

4.

a. If $u = x + y^2$, $v = y + z^2$, $w = z + x^2$, Find $\frac{\partial x}{\partial u}$. **(6)**

b. Prove that $\cos\left[i\log\left(\frac{a+ib}{a-ib}\right)\right] = \frac{a^2-b^2}{a^2+b^2}$. **(6)**

c. Solve by the Gauss-Seidel method : (Go up to 4 iterations)
$28x+4y-z=32; 2x+17y+4z=35; x+3y+10z=24$ **(8)**

5.

a. Using complex numbers prove that $\cos^6\theta + \sin^6\theta = \frac{1}{8}(3\cos 4\theta + 5)$,
hence find 7^{th} derivative with respect to θ. **(6)**

b. Prove that $(1+x)^{1/x} = e - \frac{e}{2}x + \frac{11e}{24}x^2 - \frac{7}{16}x^3 + \ldots\ldots$ **(6)**

c. If $y = e^{m\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. **(8)**

6.

a. Are the vector's $X_1=[1,3,4,2]; X_2=[3,-5,2,6]; X_3=[2,-1,3,4]$ linearly dependent? If so express X_1 as a linear combination of the other. **(6)**

b. If $x = \sqrt{v.w}$, $y = \sqrt{u.w}$ and $z = \sqrt{u.v}$ and ϕ is a function of x,y,z then

$$x\frac{\partial\phi}{\partial x}+y\frac{\partial\phi}{\partial y}+z\frac{\partial\phi}{\partial z}=u\frac{\partial\phi}{\partial u}+v\frac{\partial\phi}{\partial v}+z\frac{\partial\phi}{\partial z}$$

(6)

c. Fit a second degree parabolic curve to the following data : **(8)**

X	1	2	3	4	5	6	7	8	9
Y	2	6	7	8	10	11	11	10	9

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...by Dr. A. K. Pathak