MODEL QUESTION PAPER (Dec 2015)
S. E. SEMESTER – III
APPLIED MATHEMATICS – III
(CMPN, INFT)

[Time: 3 hours] **[Marks: 80]**

1.

a. Find $L^{-1}\left\{\frac{1}{\left(s^2+4s+13\right)^2}\right\}$ **(5Marks)**

b. Find the orthogonal trajectories of the family of cures given by $2x-x^3+3xy^2=a,'a'$ is constant. **(5Marks)**

c. Show that the set of functions $\sin x, \sin 3x, \sin 5x, ..., \sin nx$ is orthogonal on the interval $[0, \pi/2]$ and construct orthonormal set of functions. **(5Marks)**

d. Evaluate $\int_C (y-\sin x)\,dx+(\cos x)\,dy$ where C is a triangle formed by the lines $y=0, 2x=\pi$ *and* $2x=\pi y$. **(5Marks)**

2.

a. Define Fourier sine and cosine integrals. Express the function -

f(x) = sinx $0 \le x \le \pi$

= 0 x > π as Fourier sine integral. Hence evaluate $\int_0^\infty \frac{\sin wx \sin \pi w}{1-w^2}\,dw$. **(6Marks)**

b. Show that the transformation $w=\frac{1}{z}$ maps the circle |z – 3| = 5 into the circle $\left|w+\frac{3}{16}\right|=\frac{5}{16}$. **(6Marks)**

c. $\frac{d^2y}{dt^2}+9y=18t$ where y(0) = 0 and $y\left(\frac{\pi}{2}\right)=0$. **(8Marks)**

3.

a. Obtain the expansion of $f(x)=\ell x-x^2$ as a half range sine series, $0<x<\ell$ and hence deduce that

$\frac{\pi^3}{32}=1-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+.....$ & $\sum_1^\infty \frac{1}{n^6}=\frac{\pi^6}{945}$. **(6Marks)**

b. Prove that : $\overline{F} = \left(2xyz^2\right)\hat{i} + \left(x^2z^2 + z\cos yz\right)\hat{j} + \left(2x^2yz + y\cos yz\right)\hat{k}$ is conservative. Find scalar potential ϕ such that $\overline{F} = \nabla\phi$ and hence determine the work done by $\overline{F}$ displacing an object from A(0, 0, 1) to $B\left(1, \frac{\pi}{4}, 2\right)$ along the straight line AB. **(6Marks)**

c. Find the inverse Z-transform of $\frac{1}{(z-3)(z-2)}$;

i) $|z| < 2$, ii) $2 < |z| < 3$ iii) $|z| > 3$ **(8Marks)**

4.

a. Find the directional derivative of $\phi = x^2 + y^2 + z^2$ at (1, 2, 3) in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$. **(6Marks)**

b. Obtain $L\left(erf\sqrt{t}\right)$. Hence evaluate $\int_0^\infty te^{-t^2} erf\left(t\right)dt$ **(6Marks)**

c. Determine the Fourier Series for $f\left(x\right) = \left(\frac{\pi - x}{2}\right)^2$ in $[0, 2\pi]$ and deduce that, **(8Marks)**

i) $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...$

ii) $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ...$

iii) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$

5.

a. Find analytic function f(z) if $u + v = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ **(6Marks)**

b. Find the Z-transform of a^k & $\frac{a^k}{k!}, k \geq 0$ **(6Marks)**

c. Using Gauss divergence theorem evaluate

$\iint_C xz^2 dydz + \left(x^2y - z^3\right)dzdx + \left(2xy + y^2z\right)dxdy$

where S is the surface enclosing the region bounded by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and the plane z = 0. **(8Marks)**

6.

a. Find complex form of Fourier series for $f\left(x\right) = e^{ax}, -\pi < x < \pi$. **(6Marks)**

b. Find $L^{-1}\left\{\tan^{-1}\frac{2}{s^2}\right\}$ **(6Marks)**

c. Find a bilinear transformation which maps the points z = 1, i, –1 into the points w = i, 0, –i. Hence find the image of |Z| < 1. Also find fixed points. **(8Marks)**

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...by Dr. A. K. Pathak